

Queen Mary, University of London
MAS317 Linear Algebra II, 2007
Coursework 3

This coursework should be handed in by 1.30 pm on MONDAY October 15th in the orange box on the second floor in the mathematics building. Late coursework will not be accepted. Write your name and student number at the top and clip all the sheets together.

1. **[20 marks]** Let v_1, v_2, v_3, v_4 be vectors in a vector space V over $\mathbb{R}$.
Give the definition of $\langle v_1, v_2, v_3, v_4 \rangle$.
Prove that $\langle v_1, v_2, v_3, v_4 \rangle = \langle v_1, 2v_1 + 5v_2, v_3, v_4 \rangle$.
Under what conditions on $a, b \in \mathbb{R}$ is it true for all v_1, v_2, v_3, v_4 that

$$\langle v_1, v_2, v_3, v_4 \rangle = \langle v_1, av_1 + bv_2, v_3, v_4 \rangle ?$$

Give a proof.

2. **[15 marks]** Let V be a vector space of dimension n . For the following facts, give very short proofs using the results in the lecture notes:
- (i) A linearly independent set containing n vectors in V is a basis.
 - (ii) If W is a subspace of V and $\dim W = n$ then $W = V$.
 - (iii) A spanning set for V containing n vectors is a basis for V .
3. **[40 marks]** Let $u_1 = (1, -3, 4, 2)$, $u_2 = (1, -4, 11, 0)$,

$$u_3 = (-1, 2, 3, -4), \quad u_4 = (2, -5, 1, 8),$$

and let $U = \langle u_1, u_2, u_3, u_4 \rangle$.

- (i) Find a basis for the subspace U by considering the row space of a suitable matrix.
 - (ii) Find a basis for $\mathbb{R}^4$ containing the basis you have found for U .
 - (iii) If a basis for U consists of some of the vectors u_1, u_2, u_3, u_4 , how many vectors will it contain? Find such a basis.
4. **[25 marks]** Let e be an elementary row operation. Which of the following will not change when e is applied to any matrix A ?
Explain your answer, using the results in the lectures, or giving an example of a matrix A where there is a change.
- (i) the row space, (ii) the row rank, (iii) the column space, (iv) the column rank.